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and to-day a hero of eternity. Thus it is with thousands of men all over the world to-day. When they tread the wine press alone, they looked to higher callings. But they not only looked to higher callings, they worked for higher things and putting their trust in their strong right arm and the power of God, they batterd down the bulwarks between them and the world of fortune and success. While the slumberers slept they planned for the morrow, for the week, for the month, for the year, yea for life here and hereafter. This is true of Dr. Loomis. Possessing high aims and a strong will he is bound to succeed in whatever he undertakes,obstacles having been and are only incentives to more vigorous action. Building on the foundation of temperance, honesty, and industry, Dr. Loomis will succeed in whatever field he may be called, his motto being *Viam Inviniam aut Faciam.*

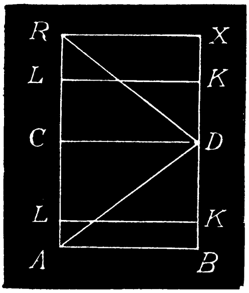
NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph. D, (Johns Hopkins): Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

CHAPTER SECOND.

[Continued from the June Number.]

PROPOSITION V. *The hypothesis of right angle, if even in a single case it is true, always in every case it alone is true.*



PROOF. Let the join CD (Fig. 4) make right angles with any two equal perpendiculars AC, BD, standing upon any other AB.

CD will be (P. III.) equal to this AB. Assume in AC, and BD produced two sects CR, DX, equal to these AC, BD; and join RX. We may easily show that the join RX will be equal to this AB, and the angles at it right. And first indeed by superposition of the quadrilateral $ABDC$ upon the quadrilateral $CDXR$, applied to the common base CD.

More elegantly then we may proceed thus. Join AD, RD. It follows (Eu I. 4.) in the triangles ACD, RCD, the bases AD, RD will be equal and likewise the angles CDA, CDR, and certainly ADB, RDX, because equal remainders from a right angle.

Whereby in turn (Eu. I. 4.) in the triangles ADB, RDX, the base AB will be equal to the base RX. Therefore (from the preceding proposition) the angles at the join RX will be right, and therefore we abide in the same hypothesis of right angle.

Since however the length of the perpendiculars can be similarly increased indefinitely, under the same base AB, the hypothesis of right angle subsisting, it is to be demonstrated that the same hypothesis will always abide in any case of diminution of those perpendiculars; which indeed is thus evinced.

Assume in AR, and BX any two equal perpendiculars AL, BK and join LK. If the angles at the join LK are not right, nevertheless they will be (P. I.) equal to each other.

Therefore they will be toward the one part, as suppose toward AB obtuse, and toward RX acute, since certainly the angles here at each of those points are equal (Eu. I. 13.) to two rights. But it also holds that the perpendiculars LR, KX, those standing on RX, will be mutually equal. Therefore (P. III.) LK will be greater indeed than the opposite RX, and less than the opposite AB.

But this is absurd; because AB, and RX have been shown equal.

Therefore the hypothesis of right angle is not changed by any diminution of the perpendiculars, whilst abides the once posited base AB.

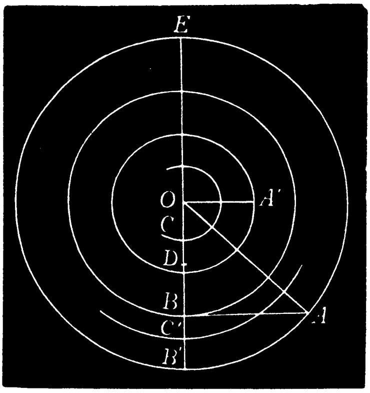
But neither is the hypothesis of right anglechanged for any diminution, or greater amplitude of the base; since manifestly may be considered as any base the perpendicular BK, or BX, and therefore may be considered in turn as perpendiculars that AB, and the equal opposite sect KL, or XR. Therefore is established that the hypothesis of right angle, if even in a single case it be true, always in every case it alone is true. Quod erat demonstrandum.

* * *

A PROOF OF THE PHYTHAGOREAN PROPOSITION.

By Professor ANDREW INGRAHAM, President of the Swain Free School, New Bedford, Massachusetts.

I. The circle whose radius is the hypothenuse is equivalent to the two circles having the base and the altitude for their respective radii.



Let the triangle OBA, rightangled at B, revolve in the plane of the paper about O. The diagram shows that the circle generated by OA is equivalent to the circle generated by OB and the annule generated by BA.

This proves the proposition to one who knows that the annule generated by BA is equivalent to the circle whose radius is BA.

II. The annule generated by a straight line perpendicular to a given line from any point in the same, while the latter revolves in their common plane around any